Table of contents

2. Calculation of earth-pressures in the GEO4 programs

Earth-pressure analysis is the stepping-stone of the GEO4 system. Stresses in soil and lateral earth pressures are analyzed in every program. This chapter explains basic compuational processes including many useful hints. Every user of GEO4 should read this chapter, regardless of which program he has bought. Following chapters related to individual programs require the basic theoretical knowledge contained in this chapter.

2.1 Stress in a soil, water uplift

Stress analysis in soil is based on existence of soil layers specified by the user during input. The program further inserts fictitious layers at the locations where the stress and lateral pressure (GWT, points of construction, etc.) change. The normal stress in the i^{th} layer is computed according to:

$$\sigma_i = \sum h_i \gamma_i$$

where:

h_i - thickness of the i^{th} layer [m],

γ_i - bulk weight of soil [kN/m^3]

If the layer is found below the ground water table, the bulk weight of a soil under the water must be defined with respect to given parameters of soil (see **Fig. 2.1**) as follows:

- option "Uplift pressure 10 kN/m3":

$$\gamma_{su} = \gamma_{sat} - 10$$

- option "Compute from porosity":

$$\gamma_{su} = \frac{1-n}{\gamma_s - 10}$$

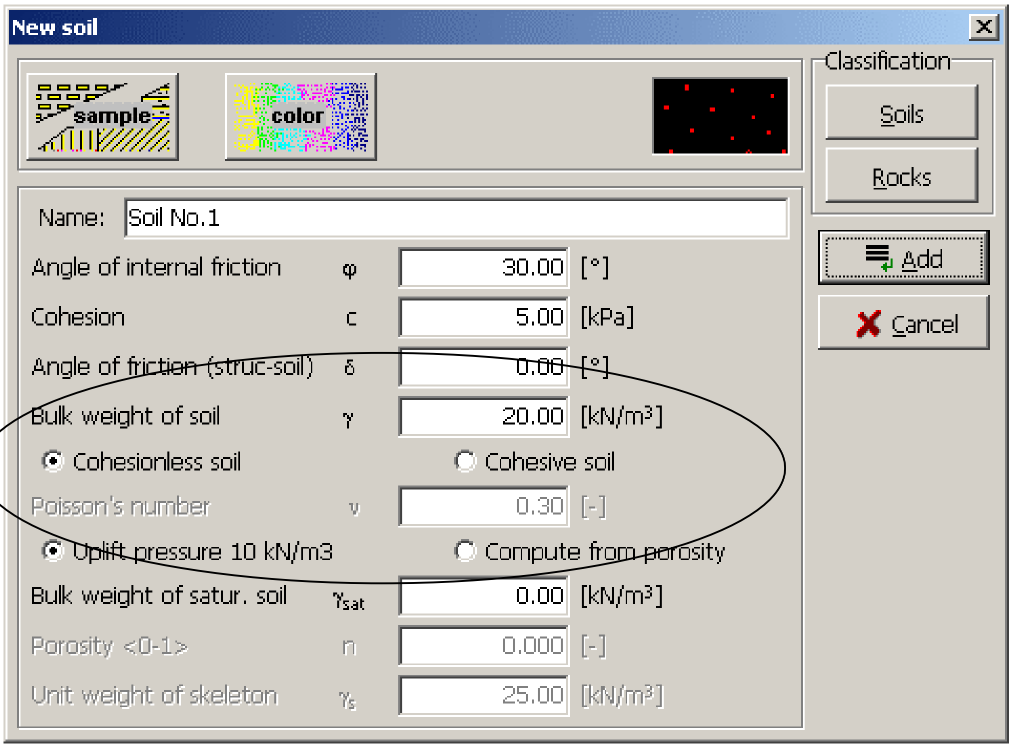


Fig. 2.1 Input of parameters of soil for computing γ_{su}

2.2 Types of earth-pressures, used notation

This part describes earth-pressures and notation used further in the guide.

Active pressure

The smallest lateral pressure developed at the onset of shear failure when the structure moves in the direction of acting earth pressure (Min. rotation of a structure to develop active earth pressure is about *2 mrad*, i.e. *2 mm/m* of structure height.)
The magnitude of active earth pressure depends on soil-structure frictional angle δ; the active pressure decreases when in increasing δ.
If structure with treated back face (foil and coatings against ground water) is analyzed one should consider a lower value of $\delta \leq 1/3\ \varphi$, for rough surfaces δ should not overcome the value $\delta = 2/3\ \varphi$.
When determining the magnitude of δ it is necessary to comply with other conditions such as equilibrium in the vertical direction. One should check, whether a structure is capable of transmitting a surcharge due to soil friction on its back without significant vertical deformations. If not one should lower δ as only a partial development of friction on back of a structure may take place. Typical structure from this point of view is an anchored wall, when there is also a surcharge in the vertical direction due components of inclined anchor forces and in most cases the value $\delta = 0$ should be used for evaluation of active earth pressure. It is always safer to consider a lower value of δ.

Pressure at rest

Pressure at rest is the smallest lateral pressure acting on an undeformable structure. It is usually considered in cases when one needs to restrict the lateral and subsequently the vertical deformation of sheeted soil (e.g., lateral strengthening of constructions during trenching ditch beneath existing foundations or during sheeting of soil with constructions sensible to nonuniform settlement in general) or if the structure loaded by earth pressure is too stiff and does not allow deformation in the loading direction required to develop an active earth pressure. In such cases it is also advisable to consider a possible implementation of so called "increased active pressure", which is a pressure between active pressure and pressure at rest. Such a pressure enters the analysis as weighted average of both pressures or as active pressure with partially mobilized angle of friction φ_{red} (between φ for active pressure and $2/3\ \varphi$ for pressure at rest-Jáky). Cohesive soils also require a reduction of cohesion as $c_{red} = c\ tg\varphi_{red} / tg\varphi$.

Passive pressure

The ultimate lateral pressure developed at the onset of shear failure of soil when the structure moves in the opposite direction than the direction of acting earth pressure. (Min. rotation required to develop passive pressure is about *2 mrad*, i.e. *2 mm/m* of structure height.) The magnitude of passive earth pressure depends on soil-structure frictional angle δ; the passive pressure increases when in increasing δ.
If structure with treated back face (foil and coatings against ground water) is analyzed one should consider a lower value of $\delta \leq 1/3\ \varphi$, for rough surfaces δ should not overcome the value $\delta = 2/3\ \varphi$. It is always safer to consider a lower value of δ.
(Most expressions follow the sign convention, according to which the values of δ, when the resultant of friction acts downwards, are negative. In the program, however, these magnitudes are put in as positive – a variant when friction acts upwards not considered in the program)

The following notation is used:

γ - bulk weight of a soil [kN/m3],
φ - angle of internal friction [^{o}],
c - cohesion of a soil [*kPa*],
α - inclination angle of back of structure from vert.d., positive - counterclockwise [^{o}],
β - inclination angle of terrain surface from vert.d., positive - counterclockwise [^{o}],
δ - angle of friction between structure and soil [^{o}],
ν - Poisson's number [-],
σ - normal stress [*kPa*].

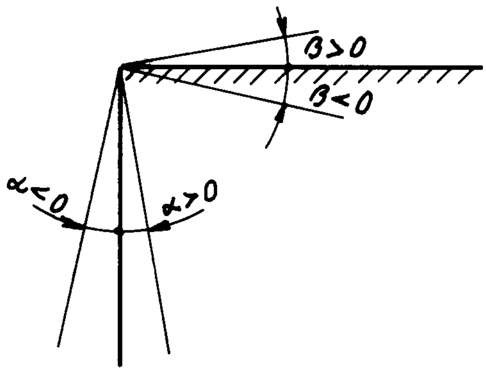


Fig. 2.2 Orientation of angles ***α a β***

2.3 Active pressure

Active earth pressure is given by

$$\sigma_a = \sigma_z K_a - 2c_{ef}\sqrt{K_{ac}}\ ,$$

where the coefficient of active pressure

$$K_a = \frac{\cos^2(\varphi - \alpha)}{\cos^2\alpha \cdot \cos(\alpha + \delta)\left[1 + \sqrt{\frac{\sin(\varphi + \delta)\cdot \sin(\varphi - \beta)}{\cos(\alpha + \delta)\cdot \cos(\alpha - \beta)}}\right]^2},$$

is an analytical expression of the Coulomb wedge method for cohesionless soils (considered for plane slip surfaces). Therefore, the angle of terrain inclination β must not overcome the design value of the angle of internal friction φ in any layer behind the back of a structure! For a homogeneous cohesionless soil this requirement is evident as it corresponds to stability condition of slope above the wall.
Horizontal and vertical components of active earth pressure are provided by

$$\sigma_{ax} = \sigma_a \cdot \cos(\alpha + \delta)$$
$$\sigma_{az} = \sigma_a \cdot \sin(\alpha + \delta).$$

The coefficient of active pressure assumes the form:

$$K_{ahc} = \frac{\cos\varphi.\cos\beta\cos(\delta-\alpha)(1+tg(-\alpha)tg\beta)}{1+\sin(\varphi+\delta-\alpha-\beta)},$$

$$K_{ac} = \frac{K_{ahc}}{\cos(\delta+\alpha)}$$

Note that tension is excluded when analyzing cohesive soils. Thus, if due to cohesion the value of active pressure becomes negative, or according to severer requirements, becomes less than the "minimum dimensioning pressure", it is set to zero or replaced by the "minimum dimensioning pressure".

When computing the active earth pressure, the option „**Consider develop. of earth-pressure wedge“** can be input in some programs (gravity wall, earth pressure), providing there is an cantilever offset on the back of sheeting (foundation plate, modification to reduce earth pressure…). Then, if selected the real back of sheeting structure above this offset is replaced by the "design back of earth wedge" in form of slip surface in soil with fully mobilized angle of friction $\delta = \varphi$. The weight of earth wedge under the "design back" will be added to the loading of a structure. Otherwise, the lateral pressure will be computed for the real back with the frictional angle $\delta \leq 2/3\ \varphi$.

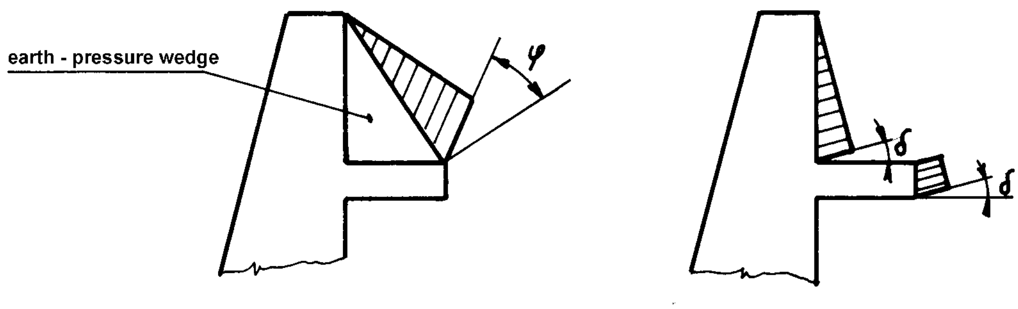


Fig. 2.3 Calculation with and without earth-pressure wedge

Another option, the **"Minimum dimensioning pressure"**, is also applicable in certain programs (Earth pressure, Sheeting, etc.). This is in support of safety of a structure in cohesive soils in regions below terrain surface, where tensile stresses are developed when computing the active pressure. In reality, however, they cannot be transmitted on a sheeting structure (consequence of separation soil due to technology of construction, isolating and drainage layer).

If selected the value of computed active pressure will not drop below 20% of vertical pressure ($K_a \geq 0,2$). Otherwise the analysis is carried out with excluded tension ($K_a \geq 0,0$). The first variant ($K_a \geq 0,2$) assumes, e.g., the possibility of increasing the lateral pressure as a consequence of filling the joint behind sheeting by rainwater. (The presented increase of lateral pressure does not correspond to full pore pressure. Should the full effect of pore pressure in a creak be considered, it would be necessary to use the possibility of a general pore pressure input – **see Section 2.6 Water**).

2.4 Passive pressure

Passive pressure is given by the following expression

$$\sigma_p = \sigma_z \cdot K_p \cdot \psi + 2 \cdot c \cdot \sqrt{K_p \cdot \psi} \ .$$

Coefficients K_p and ψ are determined by interpolating values obtained from tabulated values stored in the program. Based on the selected method the values of K_p (maximum) are computed using the soil-structure frictional $\delta = -\varphi$. For smaller (in absolute value) angles δ, the maximum values of K_p are reduced by coefficient ψ.

Horizontal and vertical components of passive pressure follow the expressions

$$\begin{aligned} \sigma_{px} &= \sigma_p \cdot \cos(\alpha + \delta) \\ \sigma_{pz} &= \sigma_p \cdot \sin(\alpha + \delta) \end{aligned} \ .$$

2.5 Pressure at rest

Pressure at rest is given by

$$\sigma_r = \sigma_z \cdot K_r \ ,$$

where $K_r = \frac{\nu}{1-\nu}$ (theory of elasticity) or $K_r = 1 - \sin\varphi$ (Jáky), respectively.

The first formula for computing K_r is used when analyzing cohesive soils, the second one is used for non-cohesive soils. The choice of type of soil (cohesive, non-cohesive) during the input of its parameters can influence the way of computing the pressure at rest. Even a typically non-cohesive soil (sand, gravel) must be put in as a cohesive one, if we want to compute the pressure at rest the Poisson number and vice versa.

For inclined ground behind a structure ($0° < \beta \le \varphi$), the pressure at rest reads

$$\sigma_r = \frac{\sigma_z \cdot K_r \cdot \sin\varphi \cdot \cos\beta}{\sin\varphi - \sin^2\beta} \ .$$

Assuming inclined back of a structure the pressure at rest is provided

$$\sigma_r = \sigma_z \sqrt{\sin^2\alpha + K_r^2 \cos^2\alpha} \ .$$

Normal and tangent components are

$$\sigma = \sigma_z \cdot \left(\sin^2\alpha + K_r \cdot \cos^2\alpha\right),$$

$$\tau = \sigma_z \cdot \left(1 - K_r\right) \cdot \sin\alpha \cdot \cos\alpha.$$

Angle between the wall and its normal δ is computed as

$$tg\delta = \frac{\left(1 - K_r\right) \cdot tg\alpha}{K_r + tg^2\alpha} \ .$$

2.6 Coefficients of computation of earth pressures "SETTINGS"

The "**Settings**" dialog window (**Fig.2.4**) serves for basic setup of the analysis. The window is slightly different in each program. However, the base where we choose the required magnitudes of coefficients of reliability of foundation soil, are common for all programs.

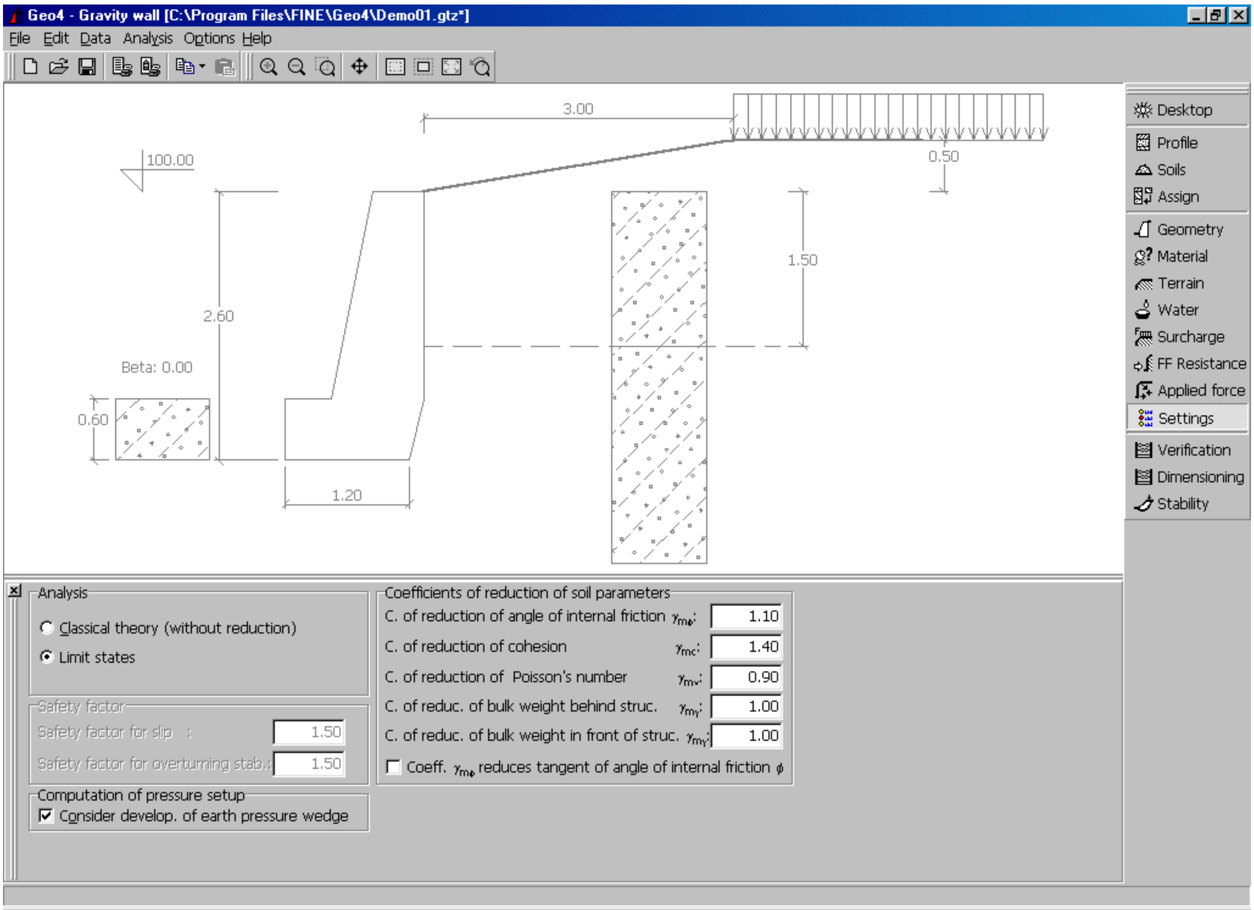


Fig. 2.4 Dialog window for setting parameters of analysis

Coefficients of reliability reduce the standard values of parameters of foundation soil to the design values:

$\gamma_{m\varphi}$ - reduces the angle of internal friction (ČSN 73 0037), or its tangent (EC7) according to the choice in the last line of the block of coefficients (at the bottom on the right hand side of subform),

γ_{mc} - reduces cohesion of soil

γ_{mv} - reduces Poisson's number (theory of elasticity is used for computation of lateral pressure at rest),

$\gamma_{m\gamma}$ - reduces the bulk weight of soil in front and behind the structure. When the analysis is carried out according to limit states, the magnitude of bulk weight of embankments and fillings either increases or decreases depending on unfavorable or favorable effects. For soils in natural state $\gamma_{m\gamma}$=1,00. In case of variable bulk weight it use reasonable to use a weighted average or a qualified estimate with respect to its influence.

Two alternatives are available for the reduction of soil parameters:

Classical theory (without reduction) – all coefficients of reliability are set to 1,00. The soil parameters are not reduced. Magnitudes are indicated with gray color – they cannot be changed. This alternative is useful when computing standard (probable) magnitudes of earth pressures.
These magnitudes can be used, e.g., for analysis of the 2^{nd} group of limit states (state of deformation) of a construction or when using "classical" methods (e.g., factor of safety, limit strength, case of „B“ EC7) in which the resulting values of characteristic stresses are reduced .

Limit states – the possibility to select coefficients according to the user requirements or standards. This option allows inserting the required magnitudes of coefficients of reliability of foundation soil.
E.g., when computing an active pressure of a non-cohesive soil with the angle of friction reduced to about 2/3, the resulting magnitude will be equal to the Jáky pressure at rest ($K_r = 1 - sin\varphi$). In analogy, the passive pressure computed with the angle of friction reduced to 2/3 will be equal to upper bound of the pressure at rest (preconsolidated soils – London clay) and when using it, the deformation needed for activation of resistance laterally pressed soil will be mobilized (in front of sheeting wall, retaining wall, horizontally loaded spread footing…).

Selecting the coefficient of reduction γ_φ from an interval *0,67* to *1* makes possible to compute magnitudes of "increased active" and "decreased passive" pressures, and thus increase reliability of a structure or limit its deformation. This coefficient must be yet multiplied by the coefficient of reliability of angle of friction $\gamma_{m\varphi}$ (providing the standard / characteristic values of pressures are not computed). The resulting coefficient of reduction $\gamma_{m\varphi}$ will be come in such a case a coefficient of two values $\gamma_\varphi . \gamma_{m\varphi}$. Analysis of the "increased active" and "decreased passive" pressures further requires reduction of cohesion as $c_{red} = c\ .tg\varphi_{red} / tg\varphi$.

2.7 Influence of terrain -"TERRAIN"

The program offers four types of a terrain behind a construction to put in – see **Fig. 2.5**

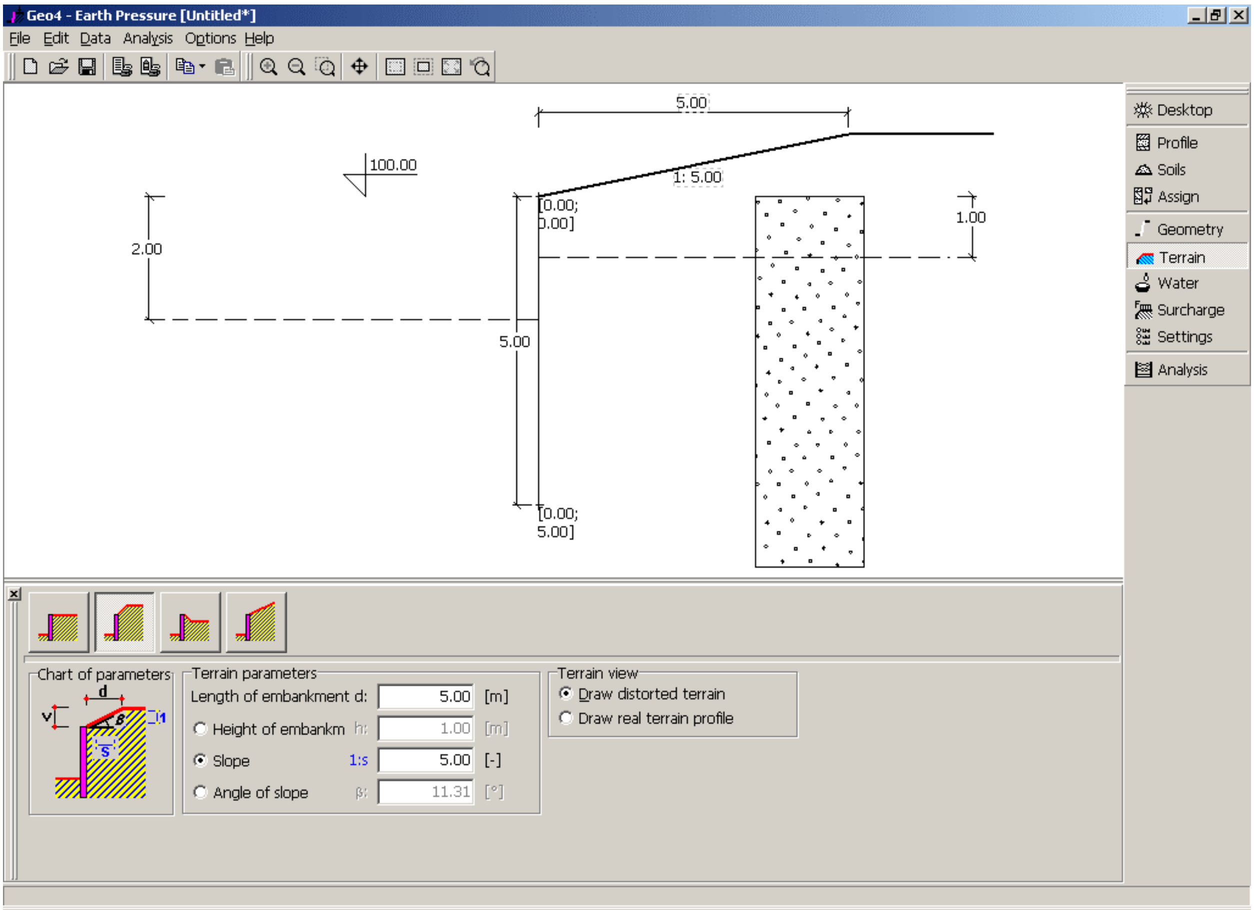


Fig. 2.5 Dialog window for inputting terrain

For a terrain creating an embankment or a cut, the resulting pressure is obtained as a maximum or minimum from pressures according to **Fig. 2.6**.

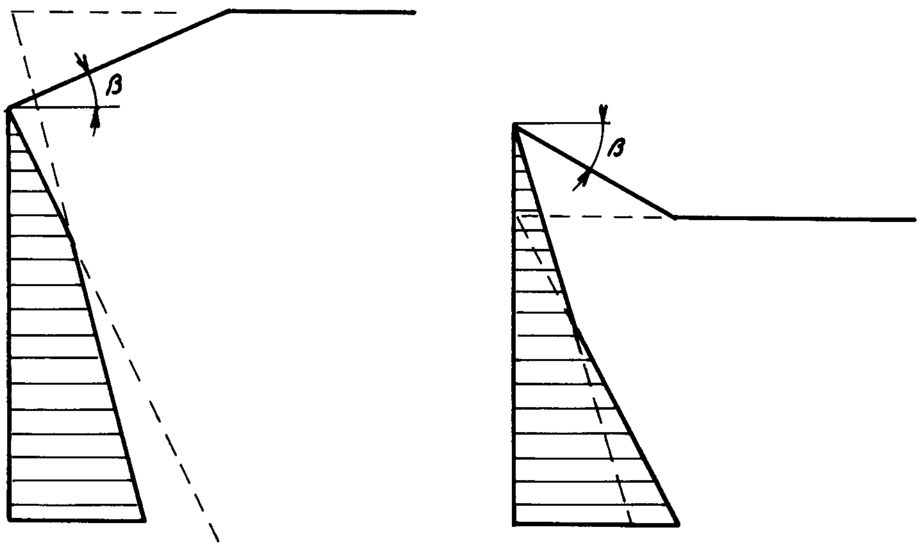


Fig. 2.6 Determination of resulting pressure for broken terrain

Angle of terrain inclination (β) is reduced, for computational purposes, as $\beta < \phi$ or $\beta < \phi_d$, respectively. This restriction applies to all layers. When exceeding the value β, analysis prompts the following information

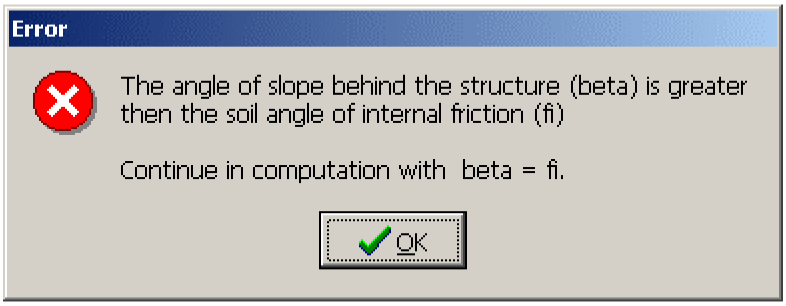


Fig. 2.7 Error message when exceeding the value β

Nevertheless, the analysis is carried out. The results, however, are not correct. Larger terrain inclinations are, therefore, necessary to model with the help of surcharge.

Example:
The terrain behind a construction was put in with inclination 45°. The smallest angle of internal friction of a soil is *17.5°*. The bulk weight of a soil in the embankment is 19 *kN/m3*. The program showed and error message during the analysis. In this case, the terrain shape will be modified according **to Fig. 2.9**.

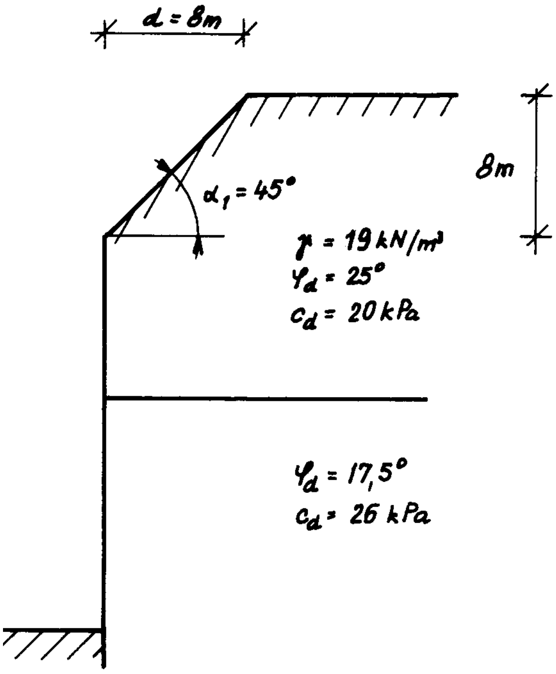


Fig. 2.8 Change of input for large terrain inclination – original input

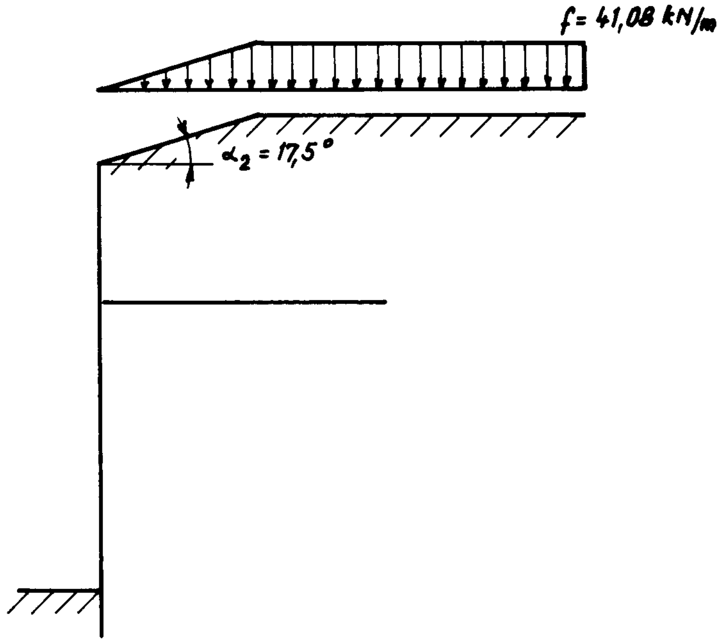


Fig. 2.9 Change of input for large terrain inclination – modified input

The surcharge is computed using the formula:

$$f = d.\gamma.(tg\alpha_1 - tg\alpha_2) = 3.20.(tg45 - tg17{,}5) = 41{,}08\ \ kN/m$$

The surcharge is a combination of trapezoidal loading (q_1=0,00 kPa, q_2=41,08 kPa, Ordinate x=0,00 m, Length=8,00 m) and strip loading (q=41,00 kPa, Ordinate x = 8,00 m, Length = at least h+d+embankment height).

2.8 Influence of water – "WATER"

The influence of groundwater (mainly the groundwater table - GWT) on a construction can be expressed using one of the variants shown in **Fig. 2.10**. Depth h_1 represents the depth of GWT behind a structure and h_2 is the depth in front of a structure.

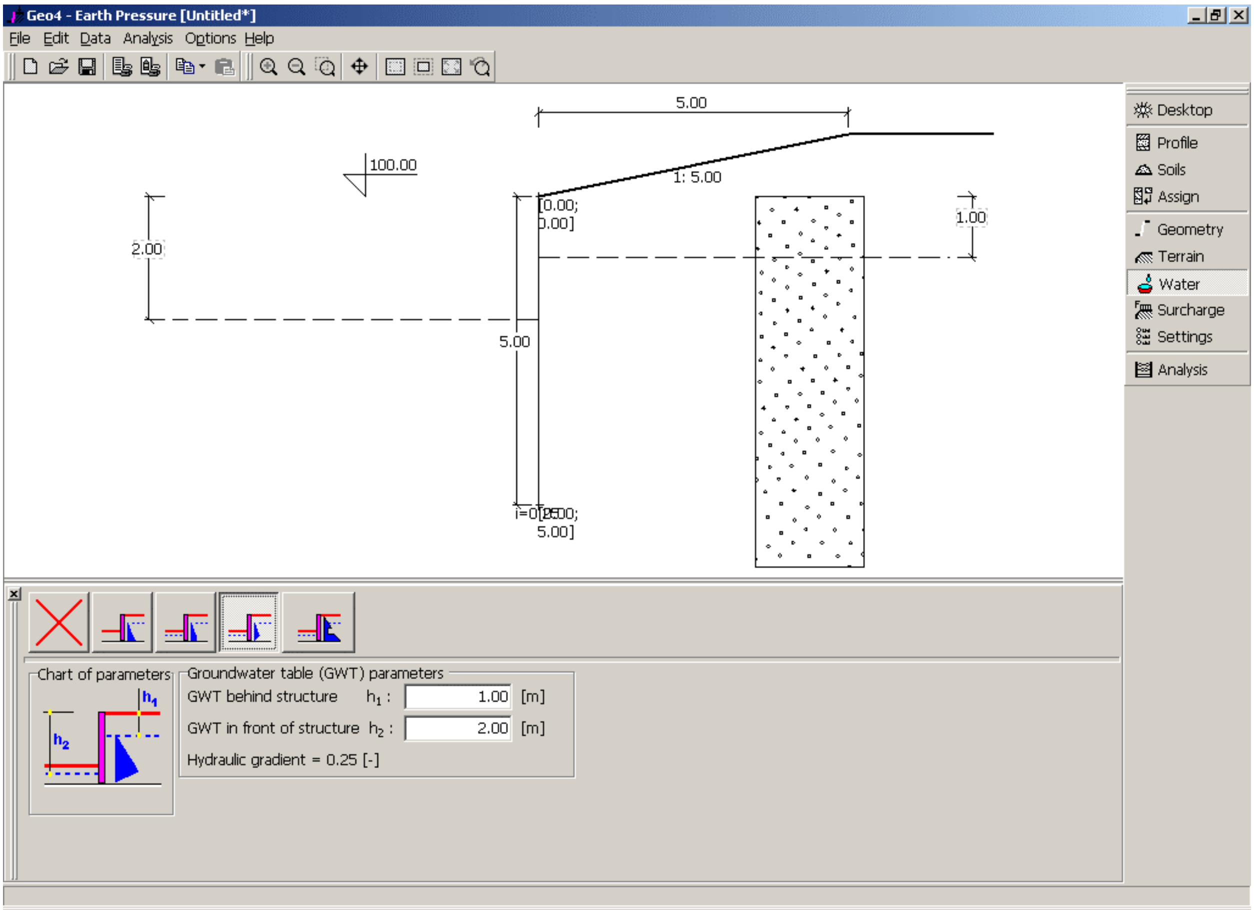
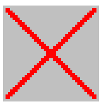


Fig. 2.10 Input of influence of water on a construction

2.8.1 Without ground water

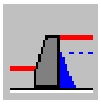


No action of ground water on a structure is considered. The ground water table is below the lowest of a structure. Subgrade soils have negligible capillary height. Therefore, neither the negative pore pressures nor an increase of bulk weight due to capillary action have to be taken into account.

Capillary action – if there are fine soils at and below the level of GWT, one should consider carefully an influence full saturation in the region of capillary action. Analysis reflects capillary action only by increased degree of saturation, and therefore γ_{sat} is inserted into parameters of soils.

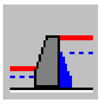
To distinguish regions with different degree of saturation, one may insert several layers of the same soil with different bulk weights. Negative pore pressures are not considered. However, for layers with different degree of saturation it is possible to use different values shear resistance influenced by suction (water and gas pore pressure difference) $u_a - u_w$.

2.8.2 Hydrostatic pressure, GWT behind structure



Heel of a structure is sunk into impermeable subsoil, and therefore the water flow below the structure is prevented. Water is found behind the back of structure only. There is no water on front face. Such a case may occur when water in front of structure flow freely due to gravity or deep drainage is used. The back of structure is loaded by hydrostatic pressure.

2.8.3 Hydrostatic pressure, GWT behind and in front of structure



Similar case as above, only loading due water in front of structure is added. This is the case when water in front of structure flows due to gravity effects or is shallowly lowered by pumping. Both the and back of structure is loaded by hydrostatic pressure (different GWTs h_1 and h_2). Dimension h_w represents difference of water tables at the back and in front structure (h_2-h_1).

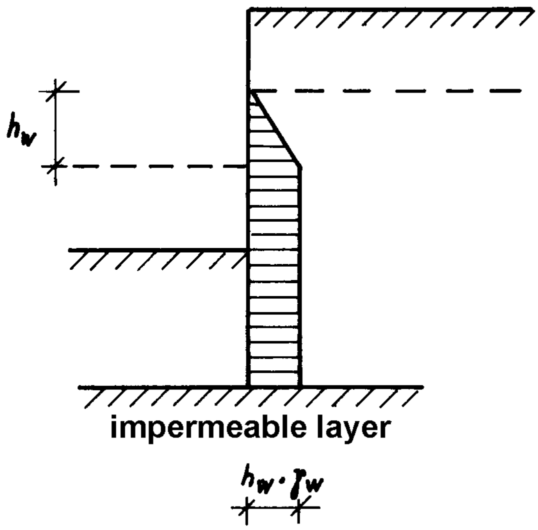
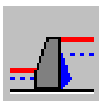


Fig. 2.11 Action of hydrostatic pressure

2.8.4 Hydrostatic pressure



Heel of a structure is sunk into permeable subsoil. This thus allows free water flow below the structure. Bulk weight of soil, lifted by uplift pressure (γ_{su}), is modified to account for flow pressure. These modifications then depend on the direction water flow.

In the analysis, the bulk weight of soil behind structure is increased by $\Delta\gamma_{su}$ and in front of structure is decreased by $\Delta\gamma_{su}$.

$$\Delta\gamma_{su} = \gamma_w \; . \; i$$

where:

i – is an average hydraulic slope [-]

$$i = \frac{h_w}{d_d + 2d_u}$$

h_w - water tables difference [m]
d_d - seepage path downwards [m]
d_u - seepage path upwards [m]

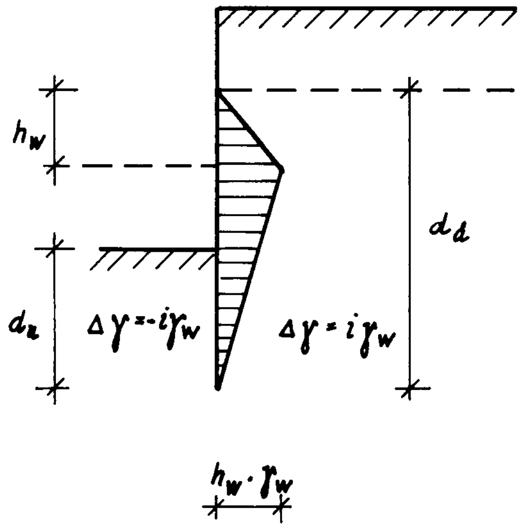


Fig. 2.12 Action of hydrostatic pressure

If $\Delta\gamma > \gamma_{su}$ then there is **leaching** in front of structure – as a consequence water flow the soil behaves weightless and cannot transmit any loading. In such case, the program prompts a message, **Fig. 2.13**, and the analysis continues γ=0. **The result therefore does not correspond to former input** – results are safer.

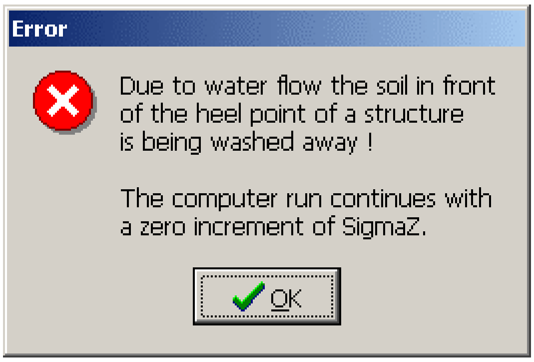
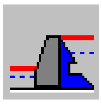


Fig. 2.13 Error message when $\Delta\gamma > \gamma_{su}$

2.8.5 Special variation of water pressure



This option allows an independent input of distribution of loading due to water at the back and in front of structure using ordinates of pore pressure at different depths. Variation of pressure between individual values is linear. At the same time it is necessary to put in levels of tables of full saturation of a soil at the back (h_1) and in front (h_2) of structure including possible decrease of bulk weight in front of structure due to water flow.

Example:
Two separated horizon lines of ground water. There are two permeable layers (sand) with one impermeable layer of clay in between, which causes separation of two hydraulic horizon lines – **Fig. 2.14**.

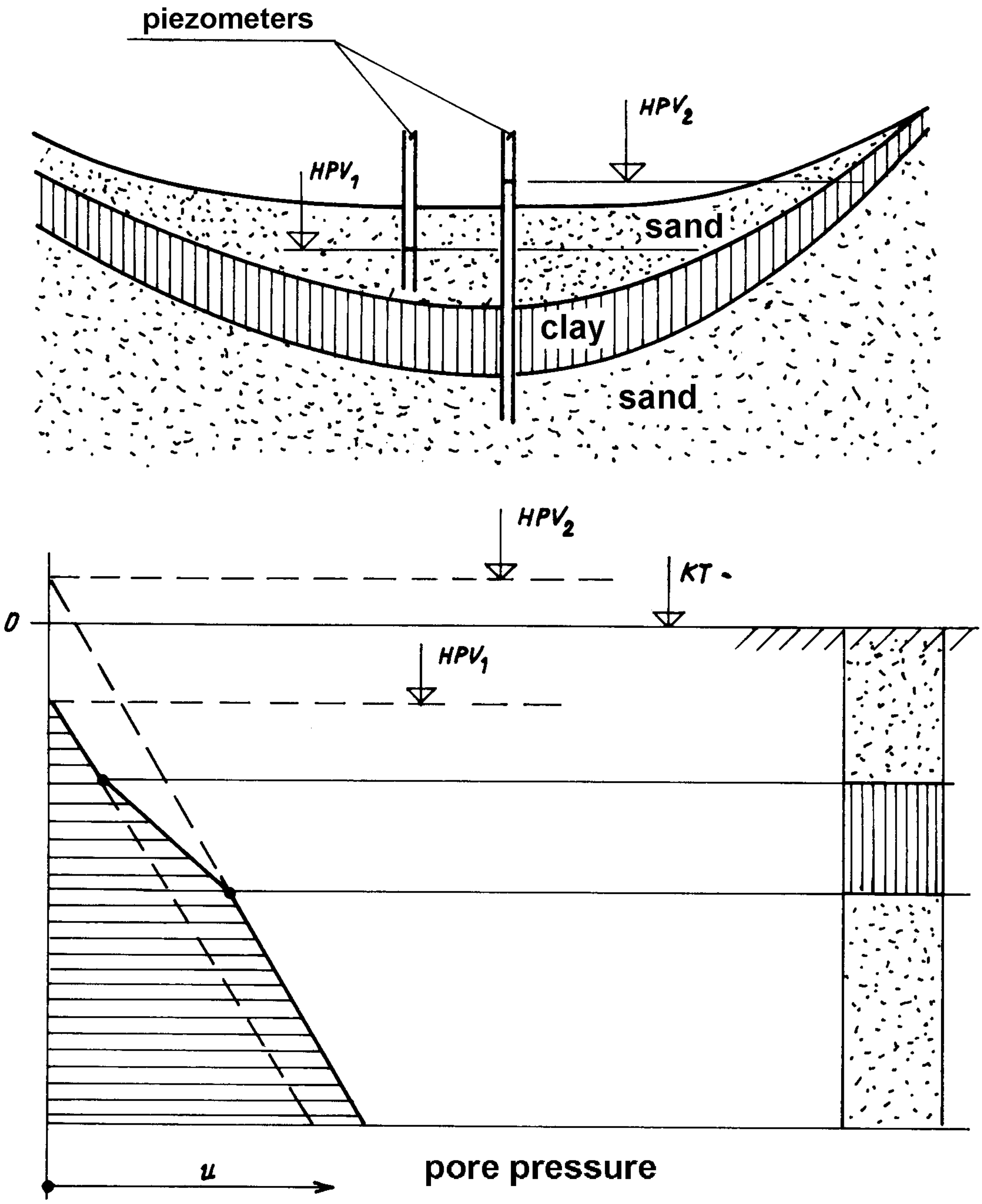


Fig. 2.14 Pore pressure distribution

Pore pressure variation above the clay layer is driven by free ground water table (**GWT_1**). Distribution of pore pressure below the clay layer results from ratio in the lower separated ground water table (**GWT_2**), where the ground water is stressed. Pore pressure distribution in clay is approximately linear.

2.9 Analysis of influence of surcharge "SURCHARGE"

An arbitrary number of surcharges behind a structure can be put in (surface, strip, trapezoidal and concentrated load). Individual surcharges can be combined as needed (e.g., to reflect complicated terrain, its larger inclination $\beta > \varphi_{d,min}$ - see example in **Section 2.7**). The used units are *kN* for concentrated and *kN/m²* for other load types.
Surcharge is defined as a surcharge of soil behind a structure – it cannot be used to surcharge on the wall crest. In such case one may use the dialog window **"Inserted forces" – see Section 3.3**.

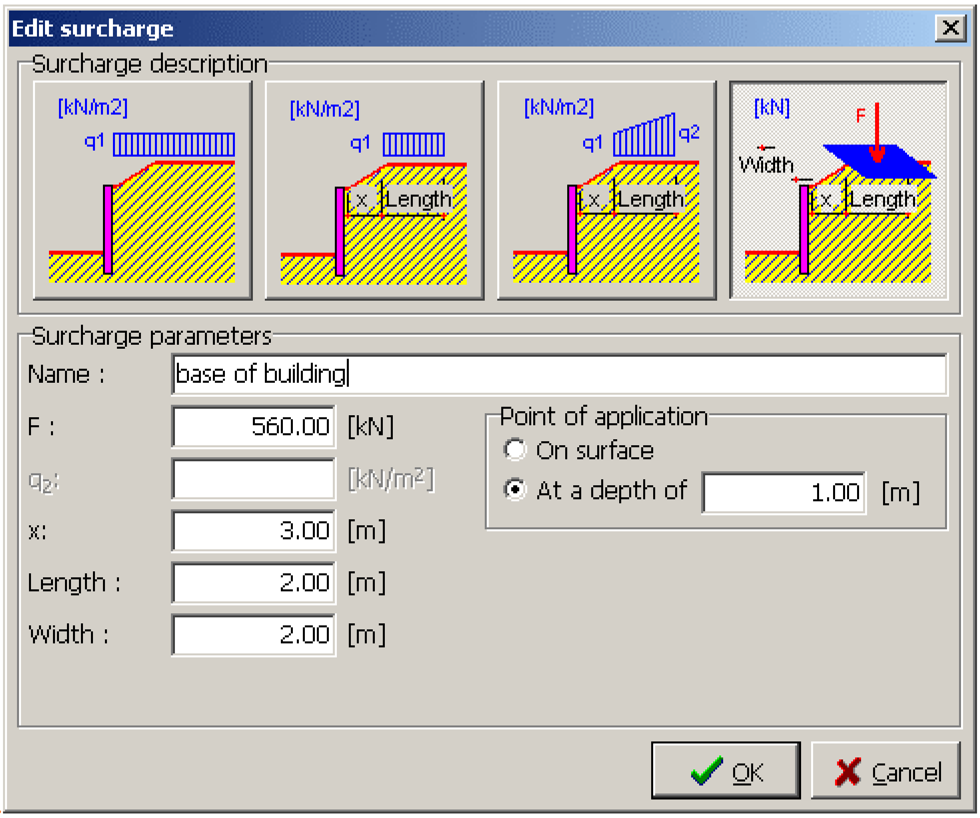
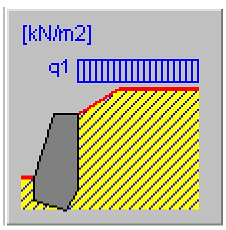


Fig. 2.15 Dialog window for input of surcharge

2.9.1 Surcharge – active pressure

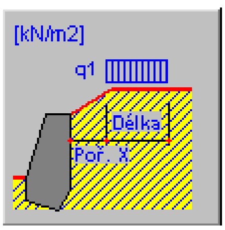


Surface surcharge is computed using to the following formula:

$$\Delta\sigma_a = f.K_a$$

where:

f - magnitude of surface load
K_a - coefficient of active pressure (**Section 2.3**)



Computation of strip is shown in **Fig. 2.16**:

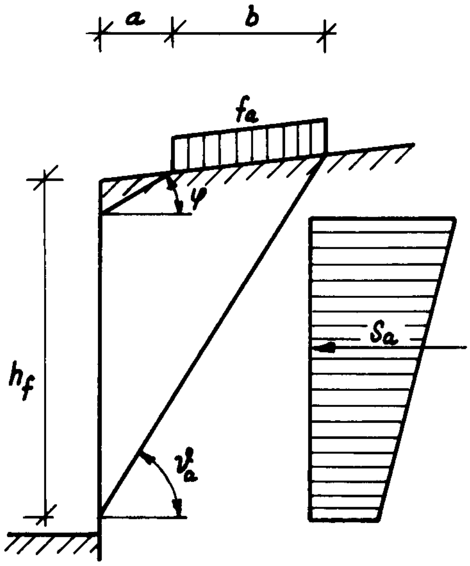


Fig. 2.16 Strip loading due to active pressure

The resultant force of active pressure S_a is given by the following formula

$$S_a = f.b.K_{af}$$

$$K_{af} = \frac{\sin(\upsilon_a - \varphi)}{\cos(\upsilon_a - \varphi - \delta)}$$

Upper and lower pressure magnitudes are provided by

$$\Delta\sigma_a' = \frac{S_a}{h_f}(1 \pm \frac{a}{a+b})$$

For nonhomogeneous soil we proceed as follows:

- compute the angle υ_a for a given soil layer
- determine the corresponding magnitude of force S_a and size of the corresponding pressure diagram
- determine the magnitude of earth pressure acting below the bottom edge of a given layer, and its ratio with respect to the overall pressure magnitude
- the surcharge is reduced using the above ratio, then the location of this surcharge on the upper edge of the subsequent layer is determined
- compute again the angle υ_a for the next layer and repeat the previous steps until the bottom of a structure is reached or the surcharge is completely exhausted

Angle υ_a is computed using:

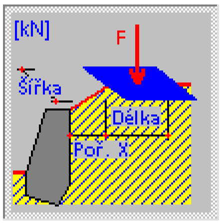
$$\upsilon_a = \varphi + \varepsilon$$

where:

$$\tan\varepsilon = \frac{\cos(\varphi-\alpha)\sin(\varphi-\beta)\cos(\alpha+\delta)+B\cos(\varphi-\beta-\alpha-\delta)}{\sin(\varphi-\alpha)\sin(\varphi-\beta)\cos(\alpha+\delta)+B\sin(\varphi-\beta-\alpha-\delta)+M}$$

$$M = \sqrt{(\sin(\varphi-\beta)\cos(\beta-\alpha)+B).(\sin(\varphi+\delta)\cos(\alpha+\delta)+B)}$$

$$B = \frac{2c\cos\alpha\cos(\beta-\alpha)\cos\varphi}{\gamma h\cos(\beta-\alpha)+\dfrac{2\sigma_z\cos\alpha\cos\beta}{\gamma h}}$$



The concentrated load is transformed into a line load (if $a<b$) or into a strip load (if $a>b$) using the following expression

$$f = \frac{F}{l+2(a+b)}$$

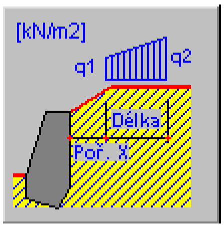
$$f_a = \frac{F}{(l+2(a+b))(a+b)}$$

The line load provided by

$$S_a = f.K_{af}$$

$$K_{af} = \frac{\sin(\upsilon_a-\varphi)}{\cos(\upsilon_a-\varphi-\delta)}$$

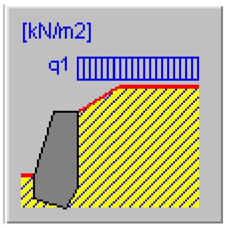
Since the pressure diagram has triangular shape of height h_f, it is easy compute the magnitudes of pressure at individual points.



The trapezoidal surcharge is subdivided in the program in 10 segments. Individual segments are treated as strip loadings. The resulting earth pressure is a sum of partial surcharges from individual segments.

2.9.2 Surcharge – pressure at rest

When computing the pressure at rest the structure is first subdivided along its height into about 30 segments, in which the program computes the magnitudes of pressure at rest due to individual surcharges. The computation itself is evident from the formulae and figures below.



The surface surcharge is given by

$$\Delta\sigma_r = f.K_r$$

where:

f - magnitude of surface load

K_r - coefficient of pressure at rest (**Section 2.5**)

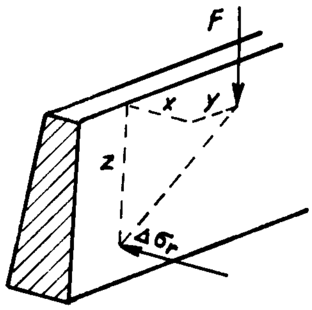
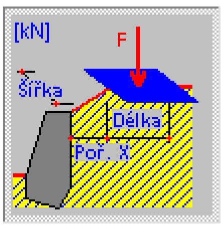


Fig. 2.17 Computation of concentrated surcharge

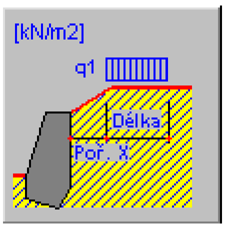


Concentrated surcharge (**Fig.2.17**):

where:

$$\Delta\sigma_r = \frac{3F}{\pi}\left(\frac{x^2 z}{r^5} + \frac{1-2\nu}{3}\left(\frac{1}{r(r+z)} - \frac{(2r+z)x^2}{(r+z)^2 x^3} - \frac{z}{r^3}\right)\right)$$

$$r = \sqrt{x^2 + y^2 + z^2}$$



Strip surcharge (**Fig.2.18**):

$$\Delta\sigma_r = \frac{f_a}{\pi}(2\alpha - \sin 2\alpha_2 + \sin 2\alpha_1)$$

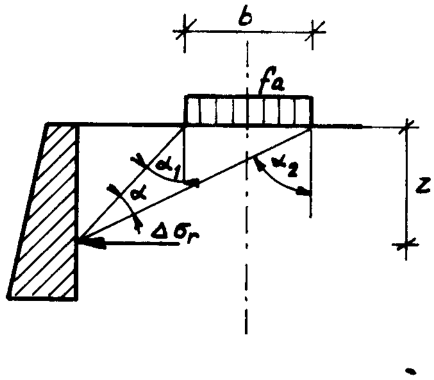
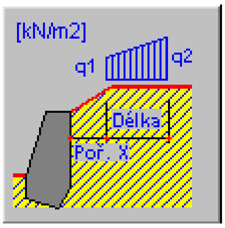
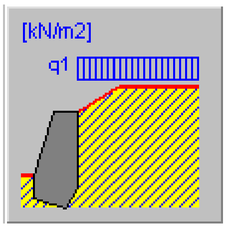


Fig. 2.18 Computation of strip loading



The trapezoidal surcharge is subdivided in the program in 10 segments. Individual segments are treated as strip loadings. The resulting earth pressure is a sum of partial surcharges from individual segments.

2.9.3 Surcharge – passive pressure



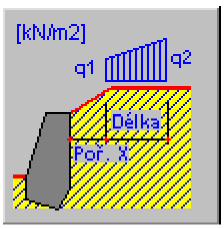
The surface surcharge is provided by

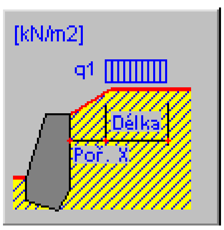
$$\Delta\sigma_p = f.K_p$$

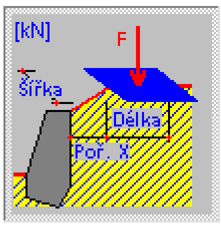
where:

f - magnitude of surface surcharge

K_p - coefficient of passive pressure (**Section 2.4**)







These types of surcharges are not considered for computation of the passive pressure.