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Appendix 2 – analysis of concrete in programs GEO4 according to EC 2

1. Materials, coefficients, notation

The following notation for material parameters is used:

f_{ck}	- characteristic cube compressive strength of concrete,
f_{cd}	- design strength of concrete in compression,
f_{cm}	- average value of design strength of concrete in tension,
$f_{ctk0,05}$	- lower bound of characteristic strength of concrete in tension,
f_{ctd}	- design strength of concrete in tension,
f_{yk}	- characteristic strength of steel,
f_{yd}	- design strength of steel in tension.

The characteristic compressive strength of concrete is the basic input parameter given by the class of concrete – it serves to derive the remaining coefficients of reliability.

$$f_{cd} = \frac{f_{ck}}{1.5}$$

$$f_{cm} = 0.3 * f_{ck}^{\frac{2}{3}}$$

$$f_{ctk0,05} = 0.7 f_{cm}$$

$$f_{ctd} = \frac{f_{ctk0,05}}{1.8}$$

$$f_{yd} = \frac{f_{yk}}{1.15}$$

In addition, a coefficient of the shape of cross-section α appears in several subsequent expressions. This coefficient ranges from the value of 0.85 for rectangular cross-sections up to the value of 0.95 for circular cross-sections.

The most common notation for geometrical parameters:

b	- cross-section width,
h	- cross-section depth,
d	- effective height of cross-section,
z	- lever arm (arm of internal forces).

2. Verification of RC rectangular cross-section under the bending moment (programs Cantilever wall, Spread footing, Abutment)

The cross-section is rectangular, reinforced on one side and loaded by the bending moment M .

The permissible moment for a given area of reinforcements A_s reads:

$$M_{rd} = \alpha \cdot 0,8 \cdot x \cdot b \cdot f_{cd} \cdot (d - 0,4x)$$

$$x = \frac{A_s \cdot f_{yd}}{0,8 \cdot \alpha \cdot b \cdot f_{cd}}$$

The program further checks whether the location of neutral axis x is less than the limit location of neutral axis x_{max} given by:

$$x_{max} = 0.45 \cdot d \quad \text{- for concrete C40/45 and lower}$$

$$x_{max} = 0.35 \cdot d \quad \text{- for concrete C45/50 and higher}$$

The computed degree of reinforcement is checked using the following expressions:

$$\rho_{min} \leq \rho \leq \rho_{max}$$

where

$$\rho = \frac{A_s}{bh}$$

$$\rho_{min} = \text{Max}\left(0,015; \frac{0,6}{f_{yk}}\right)$$

$$\rho_{max} = 0.04$$

3. Verification of spread footing for punching shear(program Spread footing)

The critical section loaded in shear (U_{cr}) is distant from the column edge by one half of the footing thickness. It is loaded by the prescribed moments M_x , M_y and by the shear force V_r provided by:

$$V_r = \frac{QA_t}{A}$$

where

- A - area of footing,
- Q - assigned vertical force developed in column,
- A_t - hatched area in fig.

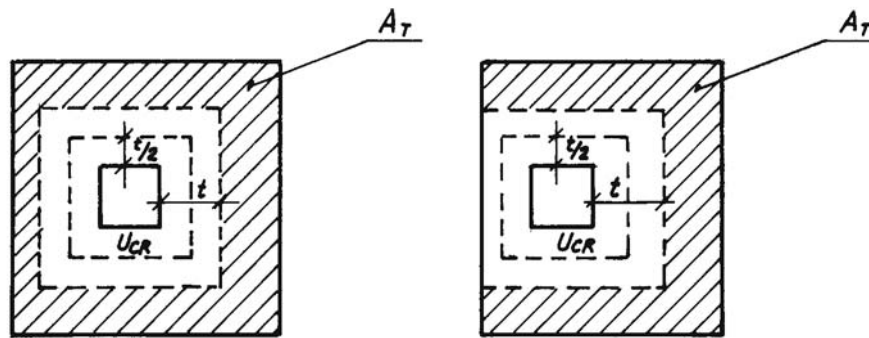


Fig. Dimensioning of shear reinforcement area A_t

The program computes the maximum shear force V_{sd} developed in the critical section, the shear force transmitted by concrete with no shear reinforcement V_{rd1} , and the maximal allowable force V_{rd2} .

$$V_{rd1} = \tau_{rd} \cdot k \cdot (1,2 + 40\rho) \cdot d$$

$$V_{rd2} = 1,6 \cdot V_{rd1}$$

where

$$\tau_{rd} = \frac{0,25 f_{ctk005}}{1,5}$$

$$k = \text{Max}(1,6 - d; 1)$$

$$\rho = \sqrt{\rho_x \rho_y}$$

ρ is the degree of reinforcement of the foundation longitudinal reinforcement in a given direction. Subsequent verification then depends on the magnitude of the maximum shear force V_{sd} .

For $V < V_c$ no shear reinforcement is needed.

For $V > V_c$ and $V_c < V_{max}$ it is necessary to design shear reinforcement. The permissible shear force is given by:

$$V_{rd3} = V_{rd1} + V_{sw}$$

$$V_{sw} = \frac{\sum A_{sw} \cdot f_{yd} \cdot \sin \alpha}{u}$$

where

- u - critical cross-section span,
- α - angle of crooks,
- A_{us} - overall area of crooks in footing.

For $V_c > V_{max}$ the shear reinforcement cannot be designed. It is therefore necessary to increase the cross-section depth.

4. Verification of cross-sections made from plain concrete (programs Gravity wall, Abutment)

The cross-section is rectangular, loaded by the bending moment M , normal force N (applied in the cross-section centroid) and by the shear force V_{sd} .

Strength of concrete cross-section loaded by the bending moment is given by:

$$M_{rd} = \frac{bh^2}{6} f_{ctd} \cdot \alpha$$

The shear strength is provided by

$$V_{rd1} = \tau_{rd} \cdot k \cdot 1,2 \cdot b \cdot h$$

where

$$\tau_{rd} = \frac{0,25 f_{ctk0,05}}{1,5}$$

$$k = \text{Max}(1,6 - h; 1)$$

Strength of concrete cross-section subject to the combination of bending moment and normal force with eccentricity e is derived from the following expressions:

$$e < 0,9a_{gc} \Rightarrow N_{rd} = x \cdot \frac{f_{cd}}{1,2} \alpha$$

$$e > 0,9a_{gc} \Rightarrow N_{rd} = \frac{b \cdot h \cdot f_{ctd} \cdot \alpha}{\frac{6e}{h} - 1}$$

kde

$$x = h - 2e$$

$$e = \frac{abs(M_{sd})}{N_{sd}}$$

$$a_{gc} = \frac{h}{2}$$

5. Verification of circular RC cross-section (program Pile)

The program verifies a reinforced concrete pile using the method of limit deformation. The maximum allowable strain of concrete in compression is 0,002 - 0.0035.

The degree of reinforcement is checked using the formula:

$$\rho_{\min} \leq \rho \leq \rho_{\max}$$

where

$$\rho = \frac{2A_s}{\pi d^2}$$

$$\rho_{\min} = \text{Max}(0,015; \frac{0,6}{f_{yk}})$$

$$\rho_{\max} = 0.04$$

where

A_s - reinforcement area
 d - pile diameter

6. Design of longitudinal reinforcement for plates (program Plate-Reinforcement)

The design of reinforcement is performed for loading caused by the bending moment M_{sd} . The program provides the required area of tensile and compressive (if needed) reinforcement. It takes into account conditions for the minimum and maximum degree of reinforcement in a given cross-section.

First, the program determines the location of neutral axis as:

$$x = \frac{d - \sqrt{d^2 - \frac{M_{sd}}{0,5 \cdot b \cdot \alpha \cdot f_{cd}}}}{0,8}$$

Providing the location of neutral axis is less than the allowable one ($x < x_{max}$), the program determines the area of tensile reinforcement A_{st} from the expression:

$$A_{st} = \frac{0,8 \cdot b \cdot x \cdot \alpha \cdot f_{cd}}{f_{yd}}$$

Providing the location of neutral axis is greater than the allowable one ($x > x_{max}$), the program determines the areas of both compressive (A_{sc}) and tensile (A_{st}) reinforcement from the expressions:

$$A_{sc} = \frac{M - F_{c,max} (d - 0,4 \cdot x_{max})}{f_{yd} \cdot z}$$

$$A_{st} = \frac{F_{c,max} + A_{sc} \cdot f_{yd}}{f_{yd}}$$

$$F_{c,max} = 0,8 \cdot x_{max} \cdot b \cdot \alpha \cdot f_{cd}$$

The limit location of neutral axis is found from:

$$x_{max} = 0,45 \cdot d \quad \text{- for concrete C40/45 and lower}$$

$$x_{max} = 0,35 \cdot d \quad \text{- for concrete C45/50 and higher}$$

The minimum area of tensile and compressive reinforcement, respectively, is provided by:

$$A_{st,min} = b \cdot h \rho_{t,min}$$

$$\rho_{t,min} = \text{Max}(0,015; \frac{0,6}{f_{yk}})$$

$$A_{sc,min} = b \cdot h \rho_{c,min}$$

$$\rho_{c,min} = 0,0015$$

If the maximum degree of tensile reinforcement ($\rho_{t,max}=0.04$) or compressive reinforcement ($\rho_{t,max}=0.08$), respectively, is exceeded, the program informs the user that the longitudinal reinforcement cannot be designed for a given cross-section.

7. Design of shear reinforcement for plates (program Plate-Reinforcement)

The program allows determination of the required amount of shear reinforcement form by stirrups and crooks, respectively.

First, the program computes the limit shear force in a given section – the shear force transmitted by concrete V_{rd1} and the maximum allowable shear force V_{rd2} .

$$V_{rd1} = \tau_{rd} \cdot k \cdot (1,2 + 40\rho) \cdot b \cdot d$$

$$V_{rd2} = 0,5 \cdot v \cdot f_{cd} \cdot b \cdot 0,9 \cdot d$$

where

$$\tau_{rd} = \frac{0,25 f_{ctk005}}{1,5}$$

$$v = \text{Max}(\frac{0,7 - f_{ck}}{200}; 0,5)$$

$$k = \text{Max}(1,6 - d; 1)$$

$$\rho = \sqrt{\rho_x \rho_y}$$

where ρ is the degree of reinforcement of the plate longitudinal tensile reinforcement .

No shear reinforcement is needed providing the shear force V is less than the shear force transmitted by concrete V_{rd1} .

If the shear reinforcement V_{sd} is greater than the maximum allowable shear force V_{rd2} , then the shear force cannot be designed and it is necessary to increase the plate dimensions.

The required area of shear reinforcement formed by stirrups per 1 m^2 is found from:

$$A_{sw} = \frac{(V_{sd} - V_{rd1})}{f_{ywd} \cdot 0,9 \cdot d}$$

8. Verification of RC rectangular cross-section under the bending moment and normal compression force (program Abutment)

The cross-section is rectangular, reinforced on one side and loaded by the bending moment M and normal compression force.

The location of neutral axis is given by:

$$x = 0,5 \cdot h - e + \sqrt{(0,5 \cdot h - e)^2 + \frac{F_s \cdot (z_s + e)}{0,5 \cdot b \cdot \alpha \cdot f_{cd}}}$$

where

$$e = \frac{M}{N}$$

$$F_s = f_{yd} \cdot A_s$$

$$z_s = d - 0,5 \cdot h$$

For $x < x_{max}$ permissible normal force is given by:

$$N_{rd} = -F_s + x \cdot b \cdot \alpha \cdot f_{cd}$$

For $x > x_{max}$ the corrected location of neutral axis is found from:

$$x = x_{\text{lim}} + \text{abs}\left(M_3 \frac{0,8.d - x_{\text{lim}}}{M_4 - M_3}\right)$$

$$M_3 = F_s \cdot (e + z_s) - x_{\text{lim}} \cdot b \cdot \alpha \cdot f_{cd}$$

$$M_4 = -0,8.d \cdot x_{\text{lim}} \cdot b \cdot \alpha \cdot f_{cd} \cdot \left(e - \frac{h - 0,8.d}{2}\right)$$

Permissible normal force is given by:

$$N_{rd} = \frac{1}{e + z_s} x \cdot b \cdot \alpha \cdot f_{cd} \left(d - \frac{x}{2}\right)$$

Permissible bending moment is provided by:

$$M_{rd} = M \cdot \frac{N_{rd}}{N}$$

Compression reinforcement is not taken to account.