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Appendix 3 – analysis of concrete in programs GEO4 according to IS 456

1. Materials, coefficients, notation

The following notation for material parameters is used:

f_{ck} – characteristic cube compressive strength of concrete,
f_{cd} – design strength of concrete in compression,
f_{ctk} – characteristic strength of concrete in tension,
f_{ctd} – design strength of concrete in tension,
f_{yk} – characteristic strength of steel,
f_{yd} – design strength of steel in tension.

The characteristic compressive strength of concrete is the basic input parameter given by the class of concrete – it serves to derive the remaining coefficients of reliability.

$$f_{cd} = 0{,}67 \frac{f_{ck}}{1{,}5}$$

$$f_{ctk} = 0.7\sqrt{f_{ck}}$$

$$f_{ctd} = 0.67 \frac{f_{ctk}}{1{,}5}$$

$$E_c = 5000\sqrt{f_{ck}}$$

$$f_{yd} = \frac{f_{yk}}{1.15}$$

The most common notation for geometrical parameters:

b – cross-section width,
h – cross-section depth,
d – effective height of cross-section,
z – lever arm (arm of internal forces).

All computations are carried out according to the theory of limit states.

2. Verification of RC rectangular cross-section under the bending moment(programs Cantilever wall, Spread footing, Abutment)

The cross-section is rectangular, reinforced on one side and loaded by the bending moment M.

The permissible moment for a given area of reinforcements A_s reads:

$$M_{rd} = b.F_c.(d - 0{,}42x)$$

$$F_c = 0{,}36.f_{ck}.x$$

$$x = \frac{A_s.f_{yd}}{b.0{,}36.f_{ck}}$$

The program further checks whether the location of neutral axis x is less than the limit location of neutral axis x_{max} given by:

$$x_{\max} = 0.53d \text{ - for steel Fe 250}$$
$$x_{\max} = 0.48.d \text{ - for steel Fe 400}$$
$$x_{\max} = 0.46.d \text{ - for steel Fe 500}$$

The computed degree of reinforcement is checked using the following expressions:

$$\rho_{\min} \leq \rho \leq \rho_{\max}$$

where

$$\rho = \frac{A_s}{bh}$$

$$\rho_{\min} = \frac{0{,}85}{f_{yk}}$$

$$\rho_{\max} = 0.04$$

3. Verification of spread footing for punching shear (program Spread footing)

The critical section loaded in shear (U_{cr}) is distant from the column edge by one half of the footing thickness. It is loaded by the prescribed moments M_x, M_y and by the shear force V_r provided by:

$$V_r = \frac{QA_t}{A}$$

where

- A - area of footing,
- Q - assigned vertical force developed in column,
- A_t - hatched area in fig.

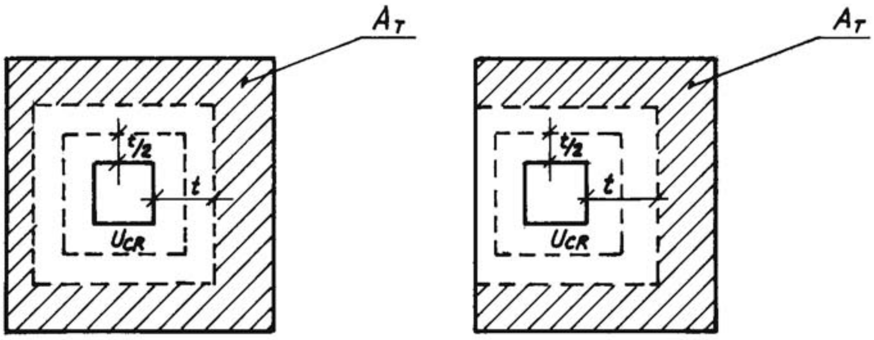

Fig. Dimensioning of shear reinforcement area A_t

The program computes the maximum shear force V developed in the critical section, the shear force transmitted by concrete with no shear reinforcement V_c, and the maximal allowable force V_{max}.

$$V_c = \tau_{rd}.k_s.h$$

$$V_{\max} = \tau_{c,\max}.h$$

where

$$\tau_c = 0{,}25\sqrt{f_{cTk}}$$

$$k_s = Min(0{,}5 + \frac{c_x}{c_y};1)$$

c_x,c_y are dimensions of footing column, $\tau_{c,max}$ is the maximum allowable shear stress in concrete listed in table 20 of the IS 456 standard.

For $V<V_c$ no shear reinforcement is needed.

For $V>V_c$ and $V_c<V_{max}$ it is necessary to design shear reinforcement. The permissable shear force is given by:

$$V_{rd3} = \frac{1}{2} V_c + V_{us}$$

$$V_{us} = \frac{\sum 0{,}87 . A_{us} . f_{yd} . \sin \alpha}{u}$$

where

u - critical cross-section span,

α - angle of crooks,

A_{us} - overall area of crooks in footing.

For $V_c>V_{max}$ the shear reinforcement cannot be designed. It is therefore necessary to increase the cross-section depth.

4. Verification of cross-sections made from plain concrete (programs Gravity wall, Abutment)

The cross-section is rectangular, loaded by the bending moment *M*, normal force *N* (applied in the cross-section centroid) and by the shear force *V*.

Strength of concrete cross-section loaded by the bending moment is given by:

$$M_{rd} = \frac{bh^2}{6} f_{ctd}$$

The shear strength is provided by

$$V_{rd} = \tau_c .b.h$$

where τ_c is the design value of stress in concrete obtained from table 19 of the IS456 standard for degree of longitudinal reinforcement ρ=0.

Strength of concrete cross-section subject to the combination of bending moment and normal force with eccentricity e is derived from the following expressions:

$$e < 0{,}9a_{gc} \;=>\; N_{rd} = x.f_{cd}$$

$$e > 0{,}9a_{gc} \;=>\; N_{rd} = \frac{b.h.f_{ctd}}{\frac{6e}{h} - 1}$$

where

$$x = h - 2e$$

$$e = \frac{abs(M)}{N}$$

$$a_{gc} = \frac{h}{2}$$

5. Verification of circular RC cross-section(program Pile)

The program verifies a reinforced concrete pile using the method of limit deformation. The maximum allowable strain of concrete in compression is 0,002 - 0.0035.

The degree of reinforcement is checked using the formula:

$$\rho_{\min} \leq \rho \leq \rho_{\max}$$

where

$$\rho = \frac{2A_s}{\pi d^2}$$

$$\rho_{\min} = \frac{0{,}85}{f_{yk}}$$

$$\rho_{\max} = 0.04$$

where

As	- reinforcement area
d	- pile diameter

6. Design of longitudinal reinforcement for plates (program Plate-Reinforcement)

The design of reinforcement is performed for loading caused by the bending moment M_{sd}. The program provides the required area of tensile and compressive (if needed) reinforcement. It takes into account conditions for the minimum and maximum degree of reinforcement in a given cross-section.

First, the program determines the location of neutral axis as:

$$x = \frac{d - \sqrt{d^2 - \frac{1.68 M_{rd}}{0.36 b f_{ck}}}}{0.84} \quad ,$$

Providing the location of neutral axis is less than the allowable one ($x<x_{max}$), the program determines the area of tensile reinforcement A_{st} from the expression:

$$A_s = \frac{0.36 b f_{ck}}{f_{yd}} x \quad .$$

Providing the location of neutral axis is greater than the allowable one ($x>x_{max}$), the program determines the areas of both compressive (A_{sc}) and tensile (A_{st}) reinforcement from the expressions:

$$A_{sc} = \frac{M - F_{c,\max}(d - 0.42.x_{\max})}{f_{yd}.z}$$

$$A_{st} = \frac{F_{c,\max} + A_{sc}.f_{yd}}{f_{yd}}$$

$$F_{c,\max} = x_{\max}.b.f_{cd}$$

The limit location of neutral axis is found from:

$x_{\max} = 0.53d$ - for steel Fe 250

$x_{\max} = 0.48.d$ - for steel Fe 400

$x_{\max} = 0.46.d$ - for steel Fe 500

The minimum area of tensile and compressive reinforcement, respectively, is provided by:

$$A_{st,\min} = b.h\rho_{t,\min}$$

$$\rho_{t,\min} = \frac{0,85}{f_{yk}}$$

$$A_{sc,\min} = b.h\rho_{c,\min}$$

$$\rho_{c,\min} = 0,0015$$

If the maximum degree of tensile reinforcement ($\rho_{t,max}$=0.04) or compressive reinforcement ($\rho_{t,max}$=0.08), respectively, is exceeded, the program informs the user that the longitudinal reinforcement cannot be designed for a given cross-section.

7. Design of shear reinforcement for plates (program Plate-Reinforcement)

The program allows determination of the required amount of shear reinforcement form by stirrups and crooks, respectively.

First, the program computes the limit shear force in a given section – the shear force transmitted by concrete V_c and the maximum allowable shear force V_{max}.

$$V_c = \tau_c .b.h$$

$$V_{\max} = \tau_{c,\max} .b.h$$

where values τ_c,$\tau_{c,max}$ are obtained from tables 19 and 20 of the IS 456 standard.

No shear reinforcement is needed providing the shear force V is less than the shear force transmitted by concrete V_c.

If the shear reinforcement V is greater than the maximum allowable shear force V_{max}, then the shear force cannot be designed and it is necessary to increase the plate dimensions.

The required area of shear reinforcement formed by stirrups per 1 m^2 is found from:

$$A_{sv} = \frac{(V - V_c)}{0{,}87.f_{ywd}.d}$$

8. Verification of RC rectangular cross-section under the bending moment and normal compression force (program Abutment)

The cross-section is rectangular, reinforced on one side and loaded by the bending moment M and normal compression force.

The location of neutral axis is given by:

$$x = 0{,}5.h - e + \sqrt{(0{,}5.h - e)^2 + \frac{F_s.(z_s + e)}{0{,}5.b.f_{cd}}}$$

where

$$e = \frac{M}{N}$$

$$F_s = f_{yd}.A_s$$

$$z_s = d - 0{,}5.h$$

For $x<x_{max}$ permissible normal force is given by:

$$N_{rd} = -F_s + x.b.f_{cd}$$

For $x>x_{max}$ the corrected location of neutral axis is found from:

$$x = x_{\max} + abs(M_3 \frac{0{,}8.d - x_{\max}}{M_4 - M_3})$$

$$M_3 = F_s.(e + z_s) - x_{\max}.b.f_{cd}$$

$$M_4 = -0{,}8.d.x_{\max}.b.f_{cd}.(e - \frac{h - 0{,}8.d}{2})$$

Permissible normal force is given by:

$$N_{rd} = \frac{1}{e + z_s} x.b.f_{cd}(d - \frac{x}{2})$$

Permissible bending moment is provided by:

$$M_{rd} = M.\frac{N_{rd}}{N}$$

Compression reinforcement is not taken to account.