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## Appendix 5 – analysis of concrete in programs GEO4 according to BS 8110

### 1. Materials, coefficients, notation

The following notation for material parameters is used:

$f_{cu}$  – characteristic cube compressive strength of concrete,

$f_y$  – characteristic strength of reinforcement,

$f_{yd}$  – design strength of steel in tension.

$$f_{yd} = 0.87 f_y$$

The characteristic compressive strength of concrete is the basic input parameter given by the class of concrete.

The most common notation for geometrical parameters:

$b$  – cross-section width,

$h$  – cross-section depth,

$d$  – effective height of cross-section,

$z$  – lever arm (arm of internal forces).

All computations are carried out according to the theory of limit states.

### 2. Verification of RC rectangular cross-section under the bending moment

(programs Cantilever wall, Spread footing, Abutment)

The cross-section is rectangular, reinforced on one side and loaded by the bending moment  $M$ .

The permissible moment for a given area of reinforcements  $A_s$  reads:

$$M_{rd} = b.F_c.(d - 0,45x)$$

$$F_c = 0,405.f_{cu}.x$$

$$x = \frac{A_s.f_{yd}}{b.0,405.f_{cu}}$$

The program further checks whether the location of neutral axis  $x$  is less than the limit location of neutral axis  $x_{max}$  given by:

$$x_{max} = 0.5d$$

The computed degree of reinforcement is checked using the following expressions:

$$\rho_{\min} \leq \rho \leq \rho_{\max}$$

where

$$\rho = \frac{A_s}{bh}$$

$$\rho_{\max} = 0.04$$

$$\rho_{\min} = 0,0013 \quad \text{for } f_y = 460 \text{ N/mm}^2$$

$$\rho_{\min} = 0,0024 \quad \text{for } f_y = 250 \text{ N/mm}^2$$

### 3. Verification of spread footing for punching shear (program Spread footing)

The critical section loaded in shear ( $U_{cr}$ ) is distant from the column edge by one half of the footing thickness. It is loaded by the prescribed moments  $M_x$ ,  $M_y$  and by the shear force  $V_r$  provided by:

$$V = \frac{QA_t}{A}$$

where

- $A$  - area of footing,
- $Q$  - assigned vertical force developed in column,
- $A_t$  - hatched area in fig.

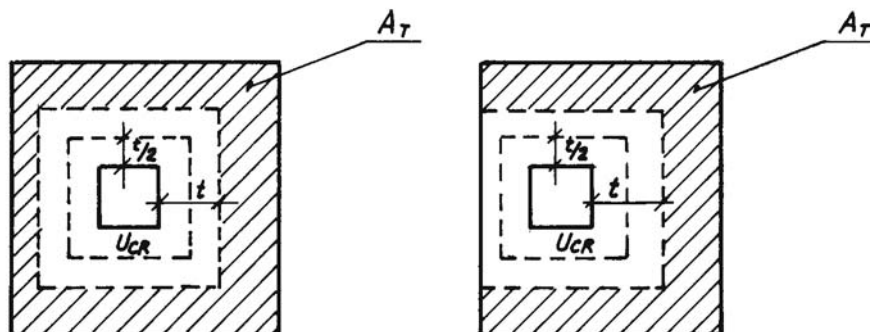


Fig. 5.16: Dimensioning of shear reinforcement area  $A_t$

The program computes the maximum shear force  $V$  developed in the critical section, the shear force transmitted by concrete with no shear reinforcement  $V_c$ , and the maximal allowable force  $V_u$ .

$$V_c = v_c \cdot d$$

$$V_u = v_u \cdot d$$

where

$v_c$  is design concrete shear stress given by following table

100As/bh	Effective depth d [mm]						
	150	175	200	225	250	300	400
0,15	0,50	0,48	0,47	0,45	0,44	0,42	0,40
0,25	0,60	0,57	0,55	0,54	0,53	0,50	0,47
0,50	0,75	0,73	0,70	0,68	0,65	0,63	0,59
0,75	0,85	0,83	0,80	0,77	0,76	0,72	0,67
1,00	0,95	0,91	0,88	0,85	0,83	0,80	0,74
1,50	1,08	1,04	1,01	0,97	0,95	0,91	0,84
2,00	1,19	1,15	1,11	1,08	1,04	1,01	0,94
3,00	1,36	1,31	1,27	1,23	1,19	1,15	1,07

The  $v_c$  values are for  $f_{cu}$  below 40N/mm<sup>2</sup> multiplied by  $(f_{cu}/40)^{1/3}$

$v_u$  is ultimate shear stress -  $v_u = 0.8\sqrt{f_{cu}}$  or 5 N/mm<sup>2</sup>

For  $V < V_c$  no shear reinforcement is needed.

For  $V > V_c$  and  $V_c < V_u$  it is necessary to design shear reinforcement. The permissible shear force is given by:

$$V_{rd} = V_c + V_{us}$$

$$V_{us} = \frac{\sum 0,87 \cdot A_{us} \cdot f_{yd} \cdot \sin \alpha}{u}$$

where

- $u$  - critical cross-section span,
- $\alpha$  - angle of crooks,
- $A_{us}$  - overall area of crooks in footing.

For  $V > V_u$  the shear reinforcement cannot be designed. It is therefore necessary to increase the cross-section depth.

#### **4. Verification of cross-sections made from plain concrete (programs Gravity wall, Abutment)**

The cross-section is rectangular, loaded by the bending moment  $M$ , normal force  $N$  (applied in the cross-section centroid) and by the shear force  $V$ .

Strength of concrete cross-section subject to the combination of bending moment and normal force with eccentricity  $e$  is derived from the following expressions:

$$e < 0,9a_{gc} \Rightarrow N_{rd} = x \cdot 0,45 \cdot f_{cu}$$

$$e > 0,9a_{gc} \Rightarrow N_{rd} = 0$$

where

$$x = h - 2e$$

$$e = \frac{\text{abs}(M)}{N}$$

$$a_{gc} = \frac{h}{2}$$

The shear strength is provided by

$$V_{rd} = v_c \cdot b \cdot h$$

where  $v_c$  is the design value of shear stress in concrete for degree of longitudinal reinforcement  $\rho=0$  –see chapter 3.

#### **5. Verification of circular RC cross-section (program Pile)**

The program verifies a reinforced concrete pile using the method of limit deformation. The maximum allowable strain of concrete in compression is 0,002 - 0.0035.

The degree of reinforcement is checked using the formula:

$$\rho_{\min} \leq \rho \leq \rho_{\max}$$

where

$$\rho = \frac{2A_s}{\pi d^2}$$

$$\rho_{\max} = 0.04$$

$$\rho_{\min} = 0,0013 \quad \text{for } f_y = 460 \text{ N/mm}^2$$

$$\rho_{\min} = 0,0024 \quad \text{for } f_y = 250 \text{ N/mm}^2$$

where

$A_s$  - reinforcement area

$d$  - pile diameter

## 6. Design of longitudinal reinforcement for plates (program Plate-Reinforcement)

The design of reinforcement is performed for loading caused by the bending moment  $M_{sd}$ . The program provides the required area of tensile and compressive (if needed) reinforcement. It takes into account conditions for the minimum and maximum degree of reinforcement in a given cross-section.

First, the program determines the location of neutral axis as:

$$x = \frac{d - \sqrt{d^2 - \frac{1.8 M_{rd}}{0.405 b f_{cu}}}}{0.9} ,$$

Providing the location of neutral axis is less than the allowable one ( $x < x_{max}$ ), the program determines the area of tensile reinforcement  $A_{st}$  from the expression:

$$A_s = \frac{0.405 b f_{cu}}{f_{yd}} x .$$

Providing the location of neutral axis is greater than the allowable one ( $x > x_{max}$ ), the program determines the areas of both compressive ( $A_{sc}$ ) and tensile ( $A_{st}$ ) reinforcement from the expressions:

$$A_{sc} = \frac{M - F_{c,\max} (d - 0.45 x_{\max})}{f_{yd} \cdot z}$$

$$A_{st} = \frac{F_{c,\max} + A_{sc} \cdot f_{yd}}{f_{yd}}$$

$$F_{c,\max} = 0,405 \cdot x_{\max} \cdot b \cdot f_{cu}$$

The limit location of neutral axis is found from:

$$x_{\max} = 0.5d$$

The minimum area of tensile and compressive reinforcement, respectively, is provided by:

$$A_{\min} = b \cdot h \rho_{\min}$$

$$\rho_{\min} = 0,0013 \quad \text{for } f_y = 460 \text{ N/mm}^2$$

$$\rho_{\min} = 0,0024 \quad \text{for } f_y = 250 \text{ N/mm}^2$$

If the maximum degree of tensile reinforcement ( $\rho_{t,\max}=0.04$ ) or compressive reinforcement ( $\rho_{c,\max}=0.04$ ), respectively, is exceeded, the program informs the user that the longitudinal reinforcement cannot be designed for a given cross-section.

## 7. Design of shear reinforcement for plates (program Plate-Reinforcement)

The program allows determination of the required amount of shear reinforcement form by stirrups and crooks, respectively.

First, the program computes the limit shear force in a given section – the shear force transmitted by concrete  $V_c$  and the maximum allowable shear force  $V_u$ .

$$V_c = v_c \cdot d$$

$$V_u = v_u \cdot d$$

where

$v_c$  is design concrete shear stress given by following table

100As/bh	Effective depth d [mm]						
	150	175	200	225	250	300	400
0,15	0,50	0,48	0,47	0,45	0,44	0,42	0,40
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0,50	0,75	0,73	0,70	0,68	0,65	0,63	0,59
0,75	0,85	0,83	0,80	0,77	0,76	0,72	0,67
1,00	0,95	0,91	0,88	0,85	0,83	0,80	0,74
1,50	1,08	1,04	1,01	0,97	0,95	0,91	0,84
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3,00	1,36	1,31	1,27	1,23	1,19	1,15	1,07
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The  $\nu_c$  values are for  $f_{cu}$  below 40N/mm<sup>2</sup> multiplied by  $(f_{cu}/40)^{1/3}$

$\nu_u$  is ultimate shear stress -  $\nu_u = 0.8\sqrt{f_{cu}}$  or 5 N/mm<sup>2</sup>

No shear reinforcement is needed providing the shear force  $V$  is less than the shear force transmitted by concrete  $V_c$ .

If the shear reinforcement  $V$  is greater than the maximum allowable shear force  $V_{max}$ , then the shear force cannot be designed and it is necessary to increase the plate dimensions.

The required area of shear reinforcement formed by stirrups per 1 m<sup>2</sup> is found from:

$$A_{sv} = \frac{(V - V_c)}{0,87 \cdot f_{yd} \cdot d}$$

## 8. Verification of RC rectangular cross-section under the bending moment and normal compression force (program Abutment)

The cross-section is rectangular, reinforced on one side and loaded by the bending moment  $M$  and normal compression force.

The location of neutral axis is given by:

$$x = 0,5 \cdot h - e + \sqrt{(0,5 \cdot h - e)^2 + \frac{F_s \cdot (z_s + e)}{0,5 \cdot b \cdot f_{cd}}}$$

where

$$e = \frac{M}{N}$$

$$F_s = f_{yd} \cdot A_s$$

$$f_{cd} = 0,45 f_{cu}$$

$$z_s = d - 0,5 \cdot h$$

For  $x < x_{lim}$  permissible normal force is given by:

$$N_{rd} = -F_s + x \cdot b \cdot f_{cd}$$

For  $x > x_{lim}$  the corrected location of neutral axis is found from:

$$x = x_{lim} + abs(M_3 \frac{0,8.d - x_{lim}}{M_4 - M_3})$$

$$M_3 = F_s \cdot (e + z_s) - x_{lim} \cdot b \cdot f_{cd}$$

$$M_4 = -0,9.d.x_{lim} \cdot b \cdot f_{cd} \cdot (e - \frac{h - 0,8.d}{2})$$

Permissible normal force is given by:

$$N_{rd} = \frac{1}{e + z_s} x \cdot b \cdot f_{cd} (d - \frac{x}{2})$$

Permissible bending moment is provided by:

$$M_{rd} = M \cdot \frac{N_{rd}}{N}$$

Compression reinforcement is not taken to account.